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Aperture Excitation of a Wire in a Rectangular Cavity

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Abstract—**The problem of determining the currents excited on a wire enclosed within a rectangular cavity is considered. The wire and cavity interior are excited by electromagnetic sources exterior to the cavity which couple to the cavity interior through a small aperture in the cavity wall. It is assumed that the wire is thin, straight, and oriented perpendicular to one of the cavity walls. An integral equation is formulated for the problem in the frequency domain using equivalent dipole moments to approximate the effects of the aperture. This integral equation is then solved numerically by the method of moments. The dyadic Green's functions for this problem are difficult to compute numerically; consequently, extensive numerical analysis is necessary to render the solution tractable. Sample numerical results are presented for representative configurations of cavity, wire, and aperture.**

I. INTRODUCTION

AN INVESTIGATION has been undertaken of the problem of a wire inside a cavity which is excited by an external source. The effects of this external source are coupled to the cavity interior and wire through an aperture in the cavity wall. The currents excited upon the wire and the fields within the cavity are to be determined. This boundary-value problem is an idealization of a wire in some metal enclosure. As examples, the wire may be inside the shielding or housing of an electronic or mechanical unit, or it might simply pass from one metal partition to another through a region which is essentially empty.

Previously, the shielding effects of infinite cylindrical structures have been treated often, i.e., [1], [2]. Recently, the problems of penetration through an aperture into a spherical cavity [3] and into a cylindrical cavity [4] also have been considered. However, the author is not aware of any previous work which treats the subsequent interaction with scatterers (such as wires) within a cavity.

II. FORMULATION OF PROBLEM

For purposes of this problem, consider a perfectly conducting rectangular cavity as shown in Fig. 1. One corner of this cavity is located at the origin of a Cartesian coordinate system. The dimensions of the cavity are denoted by a, b, and c, in the x, y, and z directions, respectively. Within this cavity, there is a perfectly conducting, round, thin wire of radius r ($r \ll \lambda$) which is assumed to be parallel to the z axis. The ends may or may not be attached to either or both walls of the cavity.

One of the walls of the cavity is perforated by a small aperture whose center is located at $\bar{r}_a = (x_a, y_a, z_a)$. The exterior region to which the aperture couples the cavity interior may be of two different types. The cavity may be located behind an infinite, perfectly conducting, planar screen such that the cavity wall containing the aperture is a portion of the infinite screen. Alternatively, the cavity

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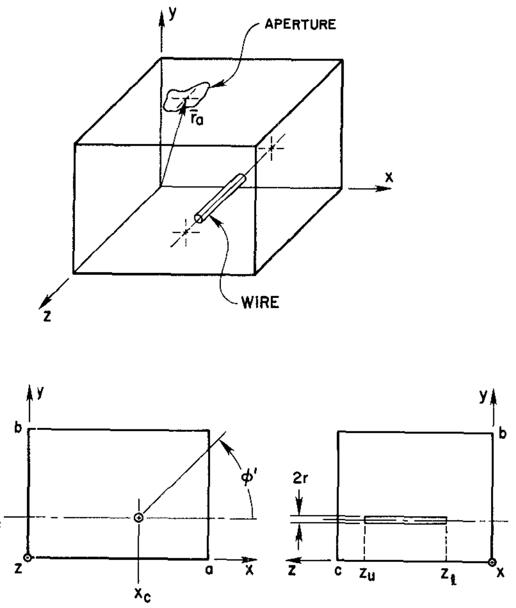

Fig. 1. Geometry for aperture-perforated rectangular cavity with interior wire.

may be situated in a free-space environment. In either case, the excitation for the problem is provided by sources in the exterior region.

Finally, it is assumed that the medium in both the interior and exterior regions is homogeneous and isotropic and is characterized by (ϵ,μ) where ϵ can be complex for a lossy medium. It is assumed that the problem is time harmonic with angular frequency ω, and the factor $e^{j\omega t}$ has been suppressed throughout.

In order to formulate an integral equation for this problem, it is necessary to know the Green's functions for the potentials and the fields within the interior, or cavity, region. These Green's functions are dyadic in nature and, as one would expect, are singular in the source region. (As a matter of notation in what follows, an uppercase G denotes a dyad due to an electric current source; similarly, a lowercase g denotes a dyad due to a magnetic current source. The subscript A, F, e, or h denotes the particular potential or field which is given by the dyad.)

The dyadic Green's function for the magnetic vector potential is defined by

$$(\nabla^2+k^2)\bar{\bar{G}}_A(\bar{r},\bar{r}')=-\bar{\bar{I}}\delta(\bar{r}-\bar{r}') \tag{1a}$$

$$\hat{n}\times\left(k^2\bar{\bar{I}}+\nabla\nabla\right)\cdot\bar{\bar{G}}_A=0, \qquad \text{on } S \tag{1b}$$

where k is the wavenumber of the homogeneous, isotropic medium of the cavity interior, $\bar{\bar{I}}$ is the identity dyad, and $\hat{n}$ is a inward-directed unit normal vector on S where S is the surface of the cavity. This Green's dyad has been derived by Tai and Rozenfeld [5] in terms of the vector wave functions $\bar{L}$, $\bar{M}$, and $\bar{N}$ and is given as a matrix by

$$\bar{\bar{G}}_A=\frac{1}{abc}\sum_{m,n,l=0}^{\infty}\frac{\epsilon_m\epsilon_n\epsilon_l}{K_{mnl}^2-k^2}\begin{Bmatrix}(cc)_x(ss)_y(ss)_z & 0 & 0\\ 0 & (ss)_x(cc)_y(ss)_z & \\ 0 & 0 & (ss)_x(ss)_y(cc)_z\end{Bmatrix} \tag{2}$$

where

$$(cc)_x(ss)_y(ss)_z=\cos k_x x\cos k_x x'\sin k_y y\cdot\sin k_y y'\sin k_z z\sin k_z z', \text{ etc.}$$

$$k_x=\frac{m\pi}{a}\quad k_y=\frac{n\pi}{b}\quad k_z=\frac{l\pi}{c}\quad K_{mnl}^2=k_x^2+k_y^2+k_z^2$$

and

$$\epsilon_i=\begin{cases}1, & i=0\\ 2, & i\neq 0.\end{cases}$$

Once $\bar{\bar{G}}_A$ has been determined, the Green's dyads for the electric and magnetic fields due to an electric current source can be found. They are defined by

$$\bar{\bar{G}}_e=\left(k^2\bar{\bar{I}}+\nabla\nabla\right)\cdot\bar{\bar{G}}_A \tag{3}$$

for the electric field and

$$\bar{\bar{G}}_h=\nabla\times\bar{\bar{G}}_A \tag{4}$$

for the magnetic field. In matrix form, $\bar{\bar{G}}_e$ and $\bar{\bar{G}}_h$ can be obtained by simply operating upon $\bar{\bar{G}}_A$ as prescribed by (3) and (4). It should be noted that this result for $\bar{\bar{G}}_e$ agrees with that derived by Tai and Rozenfeld [5] directly using the vector wave functions. It also agrees with a similar result obtained by Rahmat-Samii [6] if a minor sign error in [6, (28)] is corrected.

It now remains to determine the dyadic Green's function for the electric vector potential and its related field dyads. Consider the Green's dyad for the electric vector potential defined by

$$(\nabla^2+k^2)\bar{\bar{g}}_F(\bar{r},\bar{r}')=-\bar{\bar{I}}\delta(\bar{r}-\bar{r}') \tag{5a}$$

$$\left.\begin{aligned}\hat{n}\cdot\bar{\bar{g}}_F&=0\\ \hat{n}\times\nabla\times\bar{\bar{g}}_F&=0\end{aligned}\right\}, \qquad \text{on } S. \tag{5b}$$

Rahmat-Samii [6] has obtained a solution for $\bar{\bar{g}}_F$. It is given in matrix form by

$$\bar{\bar{g}}_F=\frac{1}{abc}\sum_{m,n,l=0}^{\infty}\frac{\epsilon_m\epsilon_n\epsilon_l}{K_{mnl}^2-k^2}\cdot\begin{Bmatrix}(ss)_x(cc)_y(cc)_z & 0 & 0\\ 0 & (cc)_x(ss)_y(cc)_z & 0\\ 0 & 0 & (cc)_x(cc)_y(ss)_z\end{Bmatrix}. \tag{6}$$

Again the sign error in [6, (26)] has been corrected.

Now that $\bar{\bar{g}}_F$ is determined, the dyads for the electric and magnetic fields due to a magnetic current source can be defined by

$$\bar{\bar{g}}_e=-\nabla\times\bar{\bar{g}}_F \tag{7}$$

and

$$\bar{\bar{g}}_h = \left(k^2\bar{\bar{I}} + \nabla\nabla\right)\cdot\bar{\bar{g}}_F. \tag{8}$$

If $\bar{\bar{g}}_e$ is written in matrix form, it is found that $\bar{\bar{g}}_e(\bar{r},\bar{r}') = -\tilde{\bar{\bar{G}}}_h(\bar{r}',\bar{r})$ where the tilde denotes the transpose of the dyad.

Before proceeding with the formulation of the integral equation, it is worthwhile to consider a few of the general properties of these dyadic Green's functions. Probably the most apparent property is that each component of each dyad is in itself a triply infinite Fourier sum. Any one of the sums can be performed analytically using one of the following relationships:

$$\sum_{m=1}^{\infty} \frac{1}{k_x^2+\alpha^2} \sin k_x x \sin k_x x' = \frac{a}{2\alpha \sinh \alpha a} \cdot \sinh \alpha x_< \sinh \alpha(a - x_>) \tag{9a}$$

$$\sum_{m=0}^{\infty} \frac{\epsilon_m}{k_x^2+\alpha^2} \cos k_x x \cos k_x x' = \frac{a}{\alpha \sinh \alpha a} \cdot \cosh \alpha x_< \cosh \alpha(a - x_>) \tag{9b}$$

$$\sum_{m=1}^{\infty} \frac{k_x}{k_x^2+\alpha^2} \cos k_x x_< \sin k_x x_> = \frac{a}{2 \sinh \alpha a} \cdot \cosh \alpha x_< \sinh \alpha(a - x_>) \tag{9c}$$

$$\sum_{m=1}^{\infty} \frac{k_x}{k_x^2+\alpha^2} \sin k_x x_< \cos k_x x_> = \frac{-a}{2 \sinh \alpha a} \cdot \sinh \alpha x_< \cosh \alpha(a - x_>) \tag{9d}$$

where

$$k_x = \frac{m\pi}{a} \qquad x_< = \min(x,x')$$

$$x_> = \max(x,x'), \qquad 0 \leqslant x \text{ and } x' \leqslant a.$$

It should be noted that there can be no doubt as to the completeness of the expansions for these dyadic Green's functions. This is because the dyads for the potentials are each comprised of the solutions to three scalar equations, for which completeness is known. The field dyads are also necessarily complete, being related to the potential dyads simply by differential operators. Term-by-term differentiation of the potential dyads as indicated by (3), (4), (7), and (8) is valid when $\bar{r} \neq \bar{r}'$ [7]. When $\bar{r} = \bar{r}'$, the series for both the potential and field dyads are necessarily divergent.

Finally, note that the effect of a differential operator on each term of any one of the sums is to introduce a multiplicative factor of m, n, or l in the numerator. This will slow the rate of convergence of the series. Thus, for $|\bar{r} - \bar{r}'| \neq 0$, components of $\bar{\bar{G}}_A$ and $\bar{\bar{g}}_F$ will exhibit the most rapid convergence, whereas $\bar{\bar{G}}_e$ and $\bar{\bar{g}}_h$, which are constructed using the second-order differential operator $\nabla\nabla\cdot$, will exhibit the slowest convergence.

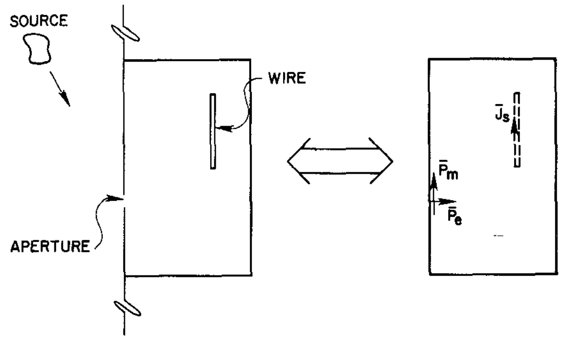

Fig. 2. Equivalent interior problem.

In order to formulate the integral equation for this problem, one applies the equivalence theorem and small-aperture theory [8]. Thus the wire is replaced by an equivalent surface current density ($\bar{J}_s$) and the aperture by equivalent electric and magnetic dipoles ($\bar{P}_e$ and $\bar{P}_m$, respectively) radiating in the presence of the shorted aperture. This equivalence is depicted in Fig. 2. Note that in this equivalent problem one has a cavity whose interior is entirely homogeneous and isotropic and is driven by the unknown sources $\bar{J}_s$, $\bar{P}_e$, and $\bar{P}_m$. Thus the fields in the cavity may be obtained by simply taking the scalar products of the appropriate dyadic Green's functions and these sources and integrating over the volume of these sources. Now the integral equation is obtained by enforcing the boundary condition that the tangential electric field must vanish on the wire surface.

Since the wire is thin, only the axial component of the wire current need be considered, which can be assumed to have negligible circumferential variation and to vanish at an unattached end of the wire. Furthermore, it is sufficient to enforce only the condition that the axial component of the electric field vanish on the wire surface. With these assumptions one obtains the integral equation

$$\frac{-j\eta}{k}\left(\frac{d^2}{dz^2} + k^2\right)\int_{z'=z_l}^{z_u} K(\bar{r},z')I_z(z')\,dz' + \hat{z}\cdot\bar{E}^i(\bar{r}) = 0 \tag{10}$$

for $\bar{r}$ on the wire surface where

$$K(\bar{r},z') = \frac{1}{2\pi}\int_{-\pi}^{\pi} G_{A_{zz}}(\bar{r},\bar{r}')\,d\phi' \tag{11}$$

and

$$\bar{E}^i(\bar{r}) = \frac{1}{\epsilon}\bar{\bar{G}}_e(\bar{r},\bar{r}_a)\cdot\bar{P}_e + jk\eta\bar{\bar{g}}_e(\bar{r},\bar{r}_a)\cdot\bar{P}_m \tag{12}$$

is the electric field produced by the aperture dipoles. In (10) $I_z(z') = 2\pi r J_z(z')$ is the wire current and $\eta = \sqrt{\mu/\epsilon}$.

Note that if (10) can be inverted, a solution for I_z will be obtained. However, it should be remembered that $\bar{E}^i$ contains $\bar{P}_e$ and $\bar{P}_m$, which still must be specified. Toward that end consider an infinite, perfectly conducting screen at $z=0$ which separates two half spaces of the same properties (μ,ϵ). This screen is perforated by a small aperture centered about the point $(0,0,0)$. If the aperture is sufficiently small and $\bar{r}$ is sufficiently far from the aperture, then the fields at $\bar{r}$ due to the aperture can be approximated by the radiation from an electric dipole with moment $\bar{P}_e$ and a magnetic dipole with moment $\bar{P}_m$ located at (0,0,0) which radiate in the presence of the unperforated screen.

The moments of the electric and magnetic dipoles for the right half space ($z>0$), which are located at $(0,0,0^+)$, are given by

$$\bar{P}_e = \epsilon\alpha_e\big(E_z^{sc-}(\bar{0}) - E_z^{sc+}(\bar{0})\big)\hat{z} \tag{13a}$$

and

$$\bar{P}_m = -\bar{\bar{\alpha}}_m\cdot\big(\bar{H}^{sc-}(\bar{0}) - \bar{H}^{sc+}(\bar{0})\big) \tag{13b}$$

where $(\bar{E}^{sc-},\bar{H}^{sc-})$ are the short-circuit fields in the left half space, that is, the fields in left half space in the presence of the unperforated screen. Similarly, $(\bar{E}^{sc+},\bar{H}^{sc+})$ are the short-circuit fields in the right half space. The electric polarizability α_e and the magnetic polarizability $\bar{\bar{\alpha}}_m$ relate the specific excitation to the moments for a given aperture. Polarizabilities are available in the literature [9]–[11] for several aperture shapes. It should be noted that, where this small-aperture theory is based upon an aperture coupling two half spaces, in the actual problem of interest the interior region is a rectangular cavity and the exterior region may or may not be a half space.

First, consider the exterior region. Suppose the cavity is behind an infinite screen, such that the exterior region is actually a half space. Then the short-circuit exterior fields can be determined from a knowledge of the incident field by application of physical optics. However, if the cavity is not behind a perfectly conducting screen, it is necessary to determine the short-circuit fields on the exterior surface of a rectangular box scatterer. This problem has been solved numerically by Tsai, Dudley, and Wilton [12]. Since the short-circuit fields are related to the surface current and charge by $\bar{J}_s = \hat{n}x\bar{H}^{sc-}$ and $q_s = \epsilon\hat{n}\cdot\bar{E}^{sc-}$, these values could also be provided by experimental measurements of surface charge and current densities. Note that $(\bar{E}^{sc-},\bar{H}^{sc-})$ and $(\bar{E}^{sc+},\bar{H}^{sc+})$ have been defined for the problem of interest to be the short-circuit fields in the exterior and interior regions, respectively. For the remainder of this paper it will be assumed that $(\bar{E}^{sc-},\bar{H}^{sc-})$ are known.

Now consider the interior region of the problem as illustrated in Fig. 2b. The fields $(\bar{E}^{sc+},\bar{H}^{sc+})$ are those which are scattered back into the aperture by the wire and the walls of the cavity. This can be ascertained by applying image theory to the cavity interior to obtain an equivalent half-space problem and then properly indentifying the imaged sources [13]. It turns out, however, that for most cases these fields are negligible when compared to the exterior short-circuit fields and consequently can be disregarded. Only in situations where the cavity is very near resonance or the aperture is very near the wire or an adjacent cavity wall might those fields have significance.

III. Numerical Solution

Now that the integral equation (10) has been obtained, one must find its solution numerically. An effective technique for obtaining such a solution is the method of moments [14]. In this method, given the linear operator equation $Lu=f$, one can approximate the unknown u by a linear combination of a finite set of expansion functions $\{u_q\}$ with unknown coefficients $\{a_q\}$. Then, by choosing a set of testing functions $\{w_p\}$ and defining an appropriate inner product, one can obtain the following matrix equation from the original operator equation:

$$\sum_{q=1}^{N}\langle w_p, Lu_q\rangle a_q = \langle w_p, f\rangle, \qquad p=1,2,\cdots,N.$$

It has been shown [15] that for integro-differential equations in the general form of (10), an efficient choice of expansion and testing functions is that of pulse functions and piecewise-sinusoidal functions, respectively. The utility of this choice is that it requires only the computation of the integral portion of the operator and thus eliminates the necessity of the subsequent computation of the differential portion of the operator.

The only difficulty remaining is the computation of the elements of the matrix equation. First, consider the computation of the various Green's functions in (2)–(4) and (6)–(8) when $\bar{r}\neq\bar{r}'$. As noted previously, each of these Green's functions can be reduced from a triple sum to a double sum which can be shown to be exponentially convergent for $|\bar{r}-\bar{r}'|\neq 0$. Indeed, the asymptotic series associated with any one of these exponentially convergent series is of the form

$$S^{asy} = \sum_{m,n} f(m,n)\frac{e^{-k_c|z-z'|}}{k_c^{\alpha}} \tag{14}$$

where $k_c^2=(m\pi/a)^2+(n\pi/b)^2, \alpha=0,1,2$, and $f(m,n)$ is a nonexponential function of m and n. Numerically, it is a good general rule to reduce the triple sum in such a way as to produce the double sum with the most rapid exponential convergence. Note that because this convergence goes as $|\bar{r}-\bar{r}'|$, as one attempts to make this computation nearer and nearer the source, the series will become more and more poorly convergent. Thus one should expect to reach a point such that for $|\bar{r}-\bar{r}'|$ less than some minimum value (β_0), numerical computation of the sum in this fashion becomes unfeasible.

Due to the exponential convergence in the asymptotic series, one would expect an efficient ordering of terms to be in order of increasing k_c. This takes advantage of the exponential convergence as well as the k_c in the denominator. At this point, it is useful to partition the m–n plane with successive curves [16]. If the partial sum of all terms lying between two successive curves is called s_q, then the double series can be converted into the single series of the form

$$S=\sum_{q=1}^{\infty} s_q.$$

By the proper selection of these curves, the most efficient ordering of terms can be determined. For this problem such a choice would be that of ellipses with semiaxes proportional to a and b. Note that for such a choice, each successive partition contains terms for which k_c is larger than in the preceding partition. Also note that since the sum of terms in the qth partition is the qth term of a single infinite series, methods used for determining the convergence of single series can be applied.

It should be noted that extensive numerical testing demonstrated that for cavities with dimensions on the order of a wavelength λ, the Green's functions could be easily obtained for values of $|\bar{r}-\bar{r}'|$ greater than $\lambda/20$ ($\beta_0 \cong \lambda/20$). Computations at even smaller values of $|\bar{r}-\bar{r}'|$ are not impossible but rather more and more time consuming.

Another computational difficulty is the evaluation of the integral of the kernel (11) over the qth pulse-expansion function of width Δ as given by

$$A_q(\bar{r}_p)=\int_{\Delta q} K(\bar{r}_p,z')\,dz'$$

where z_q is the centerpoint of the expansion function, Δq is the interval $(z_q-\frac{\Delta}{2}, z_q+\frac{\Delta}{2})$, (x_c,y_c) is the location of the center of the wire, and $\bar{r}_p=(x_c+r\cos\phi,\ y_c+r\sin\phi, z_p)$ is a point on the wire surface such that z_p is the centerpoint of the pth testing function. When $p=q$, the integrand $G_{A_{zz}}$ of the integral over the tubular wire surface segment diverges at $\bar{r}_p=\bar{r}'$. Even for $p\neq q$, if p is near q, then the integrand will converge poorly.

It is now useful to apply the reduced kernel approximation to (11), that is, assume that the current resides at the center of the wire rather than upon its surface. This approximation has been used successfully for wires in free space, and, since the wire is thin and the cavity kernel and free-space kernel differ only by a smooth, homogeneous solution to (1a), the approximation is correspondingly valid here. Note that by using this approximation, $\bar{r}_p\neq\bar{r}'$ for all $\bar{r}'$.

Using $G_{A_{zz}}$ from (2) (reducing it to a double sum by (9b)), applying the reduced kernel approximation to (11), and using a hyperbolic trigonometric identity, one can express (11) as the sum of two terms by

$$K(\bar{r}_p,z')=S(|z_p-z'|)+S(z_p+z') \tag{15}$$

where

$$S(\beta)=\frac{2}{ab}\sum_{m,n=1}^{\infty}\frac{\cosh\gamma_c(c-\beta)}{\gamma_c\sinh\gamma_c c}F(x_c,y_c)$$

$$\gamma_c^2=k_x^2+k_y^2-k^2$$

and

$$F(x_c,y_c)=\sin k_x x_c\sin k_x x_p\sin k_y y_c\sin k_y y_p.$$

Because of the exponential convergence, the series can be integrated term-by-term [7], and thus one obtains

$$A_q(\bar{r}_p)=Q(|z_p-z_q|)+Q(z_p+z_q) \tag{16}$$

where

$$Q(\alpha)=\begin{cases} P(\beta)\Big|_{\beta=\alpha-\Delta/2}^{\alpha+\Delta/2}, & \alpha\geqslant\Delta/2\\ P(\beta)\Big|_{\beta=0}^{\Delta/2+\alpha}+P(\beta)\Big|_{\beta=0}^{\Delta/2-\alpha}, & \alpha\leqslant\Delta/2\end{cases}$$

and where $P(\beta)$ is the indefinite integral of $S(\beta)$ given by

$$P(\beta)=-\frac{2}{ab}\sum_{m,n=1}^{\infty}\frac{\sinh\gamma_c(c-\beta)}{\gamma_c^2\sinh\gamma_c c}F(x_c,y_c). \tag{17}$$

Thus, if (17) can be evaluated for $\beta\geqslant 0$, then $A_q(\bar{r}_p)$ can be evaluated on the wire using the reduced kernel.

It is interesting to observe that if the centerpoints of the expansion and testing functions coincide and are uniformly spaced along the wire, (16) implies that the impedance matrix will be the sum of the two matrices, one a Toeplitz matrix and one a Hankel matrix. This has the pleasant effect of substantially reducing the matrix fill time.

Note that asymptotically P is of the following form:

$$P^{asy}\sim\sum_{m,n=1}^{\infty}F\frac{e^{-k_c\beta}}{k_c^2},\qquad 0\leqslant\beta\leqslant c.$$

Because the hyperbolic sine is an odd function, $P(2c-\beta)=-P(\beta)$. Thus for β near zero or near $2c$, poor convergence is expected. However, from a numerical standpoint there exists a $\beta_0>0$ such that for $\beta_0\leqslant\beta\leqslant 2c-\beta_0$, $P(\beta)$ can be calculated using the techniques described previously for (14).

For the special case when $\beta=0$ (or $2c$), one can analytically perform one of the sums in (17) using (9a). The resulting single sum is poorly convergent. However, its convergence can be improved by removing its asymptotic series termwise using the known sum formulation [17]

$$\sum_{n=1}^{\infty}\frac{e^{-nx}}{n}\cos n\lambda=\frac{x}{2}-\frac{1}{2}\ln(\cosh x-\cos\lambda)-\frac{1}{2}\ln 2.$$

Thus one obtains

$$P(0)=-\frac{1}{a}\sum_{m=1}^{\infty}\sin k_x x_r\sin k_x x_c \cdot\left[\frac{\sinh\gamma_b y_<\sinh\gamma_b(b-y_>)}{\gamma_b\sinh\gamma_b b}-\frac{1}{2}\frac{e^{-m\alpha}}{k_x}\right] -\frac{1}{8\pi}\ln\left(\frac{\cosh\alpha-\cos\beta_2}{\cosh\alpha-\cos\beta_1}\right)$$

where $\alpha=\pi|y_r-y_c|/a$, $\beta_1=\pi|x_r-x_c|/a$, $\beta_2=\pi(x_r+x_c)/a$, and $\gamma_b^2=k_x^2-k^2$. Numerically, this sum is rapidly convergent for nonvanishing wire radius r.

It is known that the reduced kernel (15) must contain the singular portion of the free-space reduced kernel plus a smooth homogeneous function. Thus $S(\beta)$ must also contain that singularity. The functions $\psi(z)$ and $\psi_r(z)$ are defined as the integrals of $S(\beta)$ and the singular portion of the free-space reduced kernel, respectively:

$$\psi(z)=\int_0^z S(\beta)\,d\beta=P(z)-P(0)$$

$$\psi_r(z)=\frac{1}{4\pi}\int_0^z\left[\xi^2+r^2\right]^{-1/2}d\xi=\frac{1}{4\pi}\ln\left[\frac{z}{r}+\left\{\left(\frac{z}{r}\right)^2+1\right\}^{1/2}\right].$$

If $\psi_s(z)$ is defined by $\psi_s(z)=\psi(z)-\psi_r(z)$, then ψ_s is the integral of a smooth function and thus itself is smooth.

Since $P(0)$ is readily computed, $\psi(z)$ can be computed numerically for z greater than some minimum value β_0. The function $\psi_r(z)$ can be calculated for any z. Thus $\psi_s(z)$ can be numerically evaluated for $z>\beta_0$. If ψ_s is smooth and β_0 is sufficiently small, $\psi_s(z)$ can be interpolated for $0<z<\beta_0$. Then, if $\psi_r(z)$ is added to these interpolated values of $\psi_s(z)$, $\psi(z)$ can be found for $0<z<\beta_0$.

IV. Numerical Results

In this section, selected numerical results are presented. For all results given here, it is assumed that an elliptic aperture perforates the $x=0$ wall of the cavity. This wall containing the aperture is assumed to be an infinite planar screen. Also note that all lengths are in units of wavelength λ.

First consider the case of a relatively large cavity. For this case $a=0.4$, $b=0.6$, and $c=1.3$, which is larger than the first several cavity resonances. The wire is centered in the cavity ($z_l=0.15$, $z_u=1.15$, $x_c=0.2$, $y_c=0.3$) and is one-wavelength long with radius $r=0.001$. The elliptic aperture has semiaxes of 0.05 and 0.01 in the y and z directions, respectively, and is located at $\bar{r}_a=(0, 0.2, 0.4)$. The incident plane wave impinges from the $-\bar{z}$ direction with a $-\bar{x}$-directed electric field. Since the wire is of resonant length, one would expect to excite resonant currents. Indeed, as shown in Fig. 3, this is the case.

Note that if one semiaxis of the aperture is much larger than the other, the aperture begins to look like a short slot. One would expect that the strongest coupling would occur when the slot is perpendicular to the wire and the incident electric field is perpendicular to the slot. To test this, consider a cavity with dimensions 0.7× 0.7× 0.8 ($a,b,$ and c, respectively) with a one-half wavelength wire of radius $r=0.001$ which is located in the cavity at $z_l=0.15$, $z_u=0.65$, $x_c=0.35$, and $y_c=0.49$. The aperture is located at $\bar{r}_a=(0.0, 0.4, 0.4)$, and has semiaxes of 0.07 and 0.01. The plane wave is incident from the $-\bar{x}$ direction (normal to wall of aperture) and has a $\bar{z}$-directed electric field.

Consider two cases: that where the slot is perpendicular to the incident electric field and that where the slot is parallel to the incident electric field, that is, where the major semiaxis of the aperture is in the y or z direction, respectively. Fig. 4 shows the current excited upon the wire for these two cases. It is readily seen that the current excited when the slot is perpendicular to $\bar{E}^{\mathrm{inc}}$ is approximately twenty times larger than that excited when the slot and $\bar{E}^{\mathrm{inc}}$ are parallel. For the same wire and excitation in free space [18], the current magnitude peaks at approximately 3.6 mA. Thus, even for the perpendicular slot, the shielding of the cavity reduces wire currents by approximately a factor of 350.

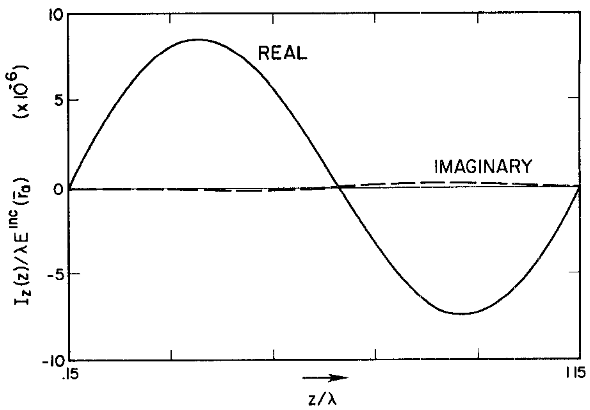

Fig. 3. Currents excited on 1-λ wire in 0.4-λ× 0.6-λ× 1.3-λ cavity.

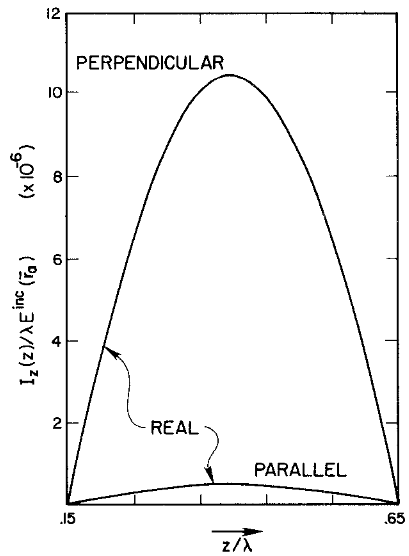

Fig. 4. Currents excited on 0.5-λ wire for slot perpendicular and parallel to $\bar{z}$- directed incident electric field.

Finally, consider the case of a wire connected to the cavity at one end. Fig. 5 shows the wire currents for this case. The cavity size is 0.7× 0.8× 0.8, and the wire axis is at $(x_c,y_c)=(0.15, 0.5)$ with a wire radius $r=0.001$. The aperture is located at $\bar{r}_a=(0.0, 0.3, 0.6)$, and its semiaxis in the y and z directions are 0.07 and 0.01, respectively. The plane wave is normally incident upon the aperture from the $-\bar{x}$ direction, and the electric field is $\bar{z}$ directed. The wire is connected at $z=0.8$, and, as expected, the axial (z) derivative of the current goes to zero at the wall. Also, as expected, the current at the free end of the wire vanishes. The current magnitude is on the order of 10 μA. For a similar cavity and excitation but having a free wire (Fig. 4, perpendicular slot), the current magnitude is also of this order.

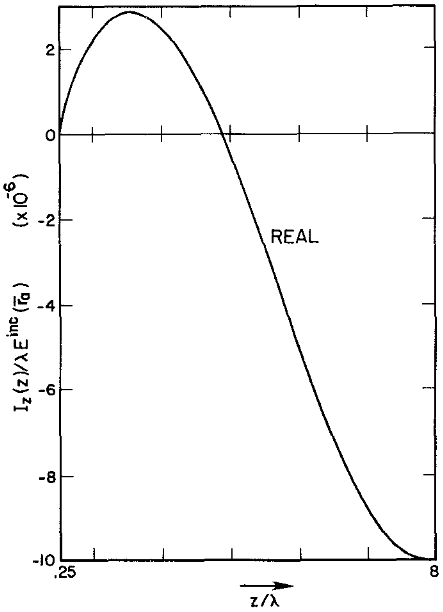

Fig. 5. Current excited on 0.55-λ wire which is attached to cavity wall at one end.

Finally, it should be noted that in Figs. 3–5 the currents were computed both with and without the contributions of $(\bar{E}^{sc+}, \bar{H}^{sc+})$. In each case, these currents were virtually indistinguishable, demonstrating the fact that $(\bar{E}^{sc+}, \bar{H}^{sc+})$ can be ignored without significant error.

V. Concluding Remarks

It has been shown that physically reasonable numerical solutions can be obtained for a variety of cavity/aperture/wire configurations. It should be noted, however, that important comparisons between theory and experiment must await the availability of experimental results applicable to this problem. Such comparisons would demonstrate the applicability of the modeling and the accuracy of the solution.

In a more general sense, this work demonstrates that the dyadic Green's functions for the cavity can be successfully treated numerically, even in or near sources. This is an important consideration in the numerical solution of an integral equation since the proper treatment of the kernel at its singularity is most crucial. These results will hopefully encourage work on similar problems within a cavity environment.

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